

Controlling Irreversibility and Directional Flow of Light with Atomic Motion

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The Doppler effect of moving atoms can create irreversibility of light. We show that the laser field in electromagnetic induced transparency (EIT) scheme with atomic motion can control the directional propagation of two counter-propagating probe fields in atomic gas cell. The quantum coherence effect serves as an optical transistor. Interference of the two output fields from the cell shows useful feature for determining the mean atomic velocity and can be useful as quantum velocimeter. We also find that the sign of the dispersive phase in EIT has a unique property, which helps to explain certain features in the interference.

In the presence of matter, the reversibility of light can be broken. An example of the broken symmetry is between the absorption and emission processes, due to the presence of spontaneous emission. Atomic motion also breaks the symmetry between absorption and stimulated emission, which is one of the ingredients for laser cooling [1]. No doubt, irreversibility of light is possible. But how can we control the irreversibility and incorporate it for useful applications? In a two-level atom, a negative detuned laser would be resonantly absorbed if the atom moves opposite to it and would be transmitted if the atom moves along it, but there is no way to control it.

In this paper, we consider an ensemble of atoms with three-level Λ or electromagnetic induced transparency (EIT) scheme moving with a velocity \vec{u} , sufficiently large such that the Doppler shift is greater than the linewidth of the excited state, $ku > \Gamma$. In each atom (Fig. 1), the same transition ($a \leftrightarrow b$) couples to *two counter-propagating* probe fields with Rabi frequencies Ω^+ and Ω^- , which are typically weaker than the control field Ω_c . The present scheme should be discerned from the velocity selective coherent population trapping (VSCPT) [2] which also has counter-propagating fields, but only one field couples to each transition.

The EIT scheme has been widely studied; mainly from the perspectives of slow light, nonlinear processes and information storage [3]. The effect of Doppler broadening on linewidth [4] and slow light [5] have been considered. However, potential application based on irreversibility of two counter-propagating probe fields in moving atoms has not been considered. Under certain controllable condition, one of the probe fields can be made transparent while the other is strongly absorbed. The combined effect of the center of mass motion and the control field in EIT provides controllable *optical rectification* to the two counter-propagating fields. Based on this mechanism, we discuss possible applications as *optical transistor* and a *quantum velocimeter*. Further analysis also yield insights on the difference between the detuning from a real level and from a level split by the control laser field.

The scheme in Fig. 1 is described by the interaction Hamiltonian $V = -\hbar[|a\rangle\langle c|\Omega_c e^{-i\nu_c t} + |a\rangle\langle b|(\Omega^+ e^{ik\hat{z}} + \Omega^- e^{-ik\hat{z}})e^{-i\nu t} + \text{adj.}]$ with the bare Hamiltonian com-

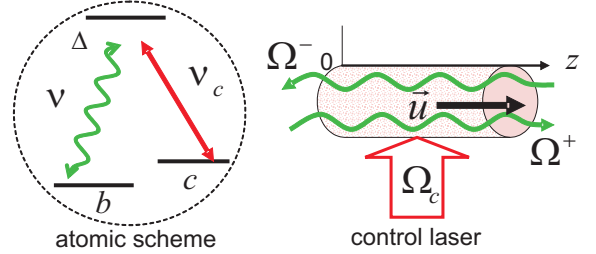


FIG. 1: (Color online) A gas with atoms with mean velocity \vec{u} in three-level EIT scheme. The transparency of the two counter-propagating probe fields is controlled by Ω_c .

posed of the kinetic energy $H_{kin} = \frac{\mathbf{p}^2}{2M}$ and atomic energy $H_a = \sum_{l=e,g_1,g_2} \hbar\omega_l |l\rangle\langle l|$. The control laser is taken to be orthogonal to the atomic beam and has no center of mass (c.m.) effect along the z -axis. Only the counter-propagating probe lasers give the c.m. effect. The center of mass position operator \hat{z} is quantized and this gives rise naturally to the first order Doppler shift $\omega_p = kp/M$ and recoil shift $\omega_r = \hbar k^2/2M$ in the transition frequency.

We have a set of infinite number of density matrix equations which contain finite coherence between different momentum families $\rho_{\alpha\beta}(p_\alpha, p_\beta) = \langle \alpha, p_\alpha | \hat{\rho} | \beta, p_\beta \rangle$ that can only be solved numerically. The presence of quantized c.m. motion makes it impossible to find exact solutions. However, we find that it is possible to obtain analytical results in the weak field limit.

For sufficiently weak probe fields as in typical EIT case, the population in the excited state is negligible. Thus it is a good approximation to disregard the momentum redistribution as the result of spontaneous emission, which gives rise to an integral terms [6] in the equations for the populations. Also, the density matrix equations may be solved analytically by truncating the set of equations based on the approximation of neglecting the coherences between two momentum families with $2\hbar k$ and larger, i.e. $\rho_{bc}(p \pm 3\hbar k, p) \ll \rho_{bc}(p \pm \hbar k, p)$, $\rho_{aa}(p \pm 2\hbar k, p) \ll \rho_{aa}(p, p) = \rho_{aa}(p)$, $\rho_{bb(cc)}(p - \hbar k, p + \hbar k) \ll \rho_{bb(cc)}(p)$. We then obtain nine equations in quasi steady state

$$T_{ab}^\pm A^\pm(p) \simeq iB^\pm(p)\Omega_c - i\Omega^\mp w_{ab}^\pm(p) \quad (1)$$

$$T_{bc}^{\pm*} B^{\pm}(p) \simeq i\Omega_c^* A^{\pm}(p) - i\Omega^{\mp} C(p) \quad (2)$$

$$T_{ac}^* C(p) \simeq i\Omega_c^* w_{ac}(p) - i\Omega^{+*} B^{-}(p) - i\Omega^{-*} B^{+}(p) \quad (3)$$

where the slowly varying coherences are

$$A^{\pm}(p, t) = e^{i(\Delta \pm \omega_p + \omega_r)t} \rho_{ab}(p, p \pm \hbar k, t), \quad (4)$$

$$B^{\pm}(p, t) = e^{i(\Delta - \Delta_c \pm \omega_p + \omega_r)t} \rho_{cb}(p, p \pm \hbar k, t) \quad (5)$$

$$C(p, t) = e^{-i\Delta_c t} \rho_{ca}(p, t) \quad (6)$$

The complex decoherences that include the Doppler and recoil shifts are $T_{ab}^{\pm} = \gamma_{ab} - i(\Delta \pm \omega_p + \omega_r)$, $T_{bc}^{\pm} = \gamma_{bc} - i(\Delta_c - (\Delta \pm \omega_p + \omega_r))$ and $T_{ac} = \gamma_{ac} - i\Delta_c$. The population inversions are defined as $w_{ac}(p) = \rho_{aa}(p) - \rho_{cc}(p)$ and $w_{ab}^{\pm}(p) = \rho_{aa}(p) - \rho_{bb}(p \pm \hbar k)$ which can be taken to be the initial values for weak probe fields $\rho_{aa}(p, t) \simeq \rho_{aa}(p, 0) = \rho_{aa} f(p)$, $\rho_{cc}(p, t) \simeq \rho_{cc}(p, 0)$ and $\rho_{bb}(p \pm \hbar k, t) \simeq \rho_{bb}(p, t) \simeq \rho_{bb}(p, 0) = \rho_{bb} f(p)$. The populations depend on the momentum distribution of a gas jet $f(p) = \exp[-(p - \bar{p})^2 / \Delta p^2]$ with $\int_{-\infty}^{\infty} f(p) dp = 1$. The gas has velocity width of $\Delta p = \sqrt{2Mk_B T}$ and a mean momentum $\bar{p} = m\bar{u}$.

The five coupled (1)-(3) can be solved exactly

$$A^{\mp}(p, z) = -i \frac{\Omega^{\pm}}{\Upsilon} [I^{\mp} (w_{ab}^{\mp} T_{ab}^{\pm} T_{bc}^{*\mp} + w_{ab}^{\pm} I_c) / J^+ J^- + w_{ab}^{\mp} I^{\pm} / J^{\mp} + (w_{ab}^{\mp} T_{ac}^* T_{bc}^{*\mp} - w_{ac} I_c) / J^{\mp}] \quad (7)$$

where $\Upsilon = T_{ac}^* + I^- T_{ab}^p / J^p + I^p T_{ab}^- / J^-$, $J^{\pm} = T_{ab}^{\pm} T_{bc}^{*\pm} + I_c$ and $I^{\pm} = |\Omega^{\pm}|^2$, $I_c = |\Omega_c|^2$.

The solutions are related to the propagation equations

$$\frac{\partial}{\partial z} \Omega^{\pm}(z) = i\kappa g \int_{-\infty}^{\infty} A^{\mp}(p, z, I^+, I^-) dp = G^{\pm} \Omega^{\pm} \quad (8)$$

where $\kappa g = N \frac{\mu_o c |\omega|^2}{2\hbar} = N \frac{\omega |\phi|^2}{2\hbar \epsilon_0 c}$. Please note that we have done the momentum integration differently from previous works [5], [4]. Note that this is a general approach, that enables the populations in different levels to take different distributions; for example: $\rho_{cc}(p) = 0.2f(p)$, $\rho_{bb}(p) = 0.8f(p - \hbar k)$, $\rho_{aa}(p) = 0$ subjected to normalization $\sum_{x=a,b,c} \int_{-\infty}^{\infty} \rho_{xx}(p) dp = 1$.

Equations (7)-(8) which imply that the variables A^{\pm} , B^{\pm} and C depend on z . The equations are highly nonlinear due to the dependence of Eq. (7) on I^{\pm} , but can be solved numerically. The nonlinearity and cross-coupling of the fields arise from the last two terms in Eq. (3). The cross-coupling correspond to two probe fields interacting simultaneously with the same atom.

In the limit of small probe fields $I^{\pm} \ll I_c$, $(T_{ab}^{\pm})^2$ the cross-interaction is weak and negligible, and we have a linear theory $A^{\pm}(p, z) = \frac{-i\Omega^{\mp}}{J^{\pm}} [w_{ab}^{\pm} T_{bc}^{*\pm} - w_{ac} I_c / T_{ac}^*]$ and Eqs. (8) yield the solutions $\Omega^{\pm}(z) = \Omega^{\pm}(0) e^{G^{\pm} z}$ where the complex 'gain' becomes

$$G^{\pm} = \kappa g \int \frac{w_{ab}^{\mp} T_{bc}^{*\mp} - w_{ac} I_c / T_{ac}^*}{J^{\mp}} dp \quad (9)$$

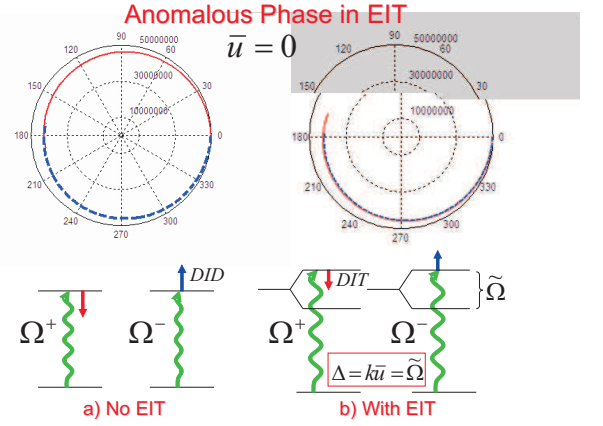


FIG. 2: (Color online) Spatial variations of the probe fields $|\Omega^+|$ (red solid line) and $|\Omega^-|$ (blue dash line) through polar plots for : a) $\Delta = 0, \Omega_c = 0$, b) $\Delta = k\bar{u}$ and finite Ω_c such that $\tilde{\Omega} (= \sqrt{\Omega_c^2 - \gamma_{ac}^2}) = \omega_D$. The upper level a is split by $2\tilde{\Omega}$. Doppler shifts of co-propagating field (red arrow) and counter-propagating field (blue arrow) lead to Doppler induced detuning (DID), Doppler induced transparency (DIT) and possibly Doppler induced resonant (DIR). We use mean velocity $\bar{u} = 300 \text{ms}^{-1}$, Gaussian width $\Delta u = \sqrt{k_B T / M}$, $T = 1 \text{K}$, $M = 87(\text{Rb})$, $\gamma_{bc} = 0.1\gamma_{ac}$ and propagation length $L = 0.1 \text{m}$.

If we neglect the superscripts \pm , we recover the known [4] relation $\tilde{\rho}_{ab} \simeq -i\Omega \frac{w_{ab} T_{bc}^* - w_{ac} I_c / T_{ac}^*}{(T_{bc}^* T_{ab} + I_c)}$.

The real parts of G^{\pm} give the absorption coefficients (if negative) and gain (if positive) while the imaginary part gives the change in the wavevector. We plot $|\Omega^{\pm}(z)| = \Omega_0 e^{\text{Re} G^{\pm} z}$ in Fig. 2 with the assumption of two-photon resonance $\Delta = \Delta_c$ and decoherence $\gamma_{bc} = 0.1\gamma_{ac}$. The polar plots illustrate the spatial evolutions of the amplitudes $|\Omega^{\pm}(z)|$ (radial lengths) and the phases, $\theta^{\pm} = \text{Im} G^{\pm} z$ (angles). In the absence of atomic motion, we find a new subtle effect of EIT. The polar plot (Fig. 2b) shows that linear responses or the susceptibilities $\chi^{(1)\pm}$ for the two fields Ω^{\pm} are the same when the control field is on, which is in contrast to the case without control field (Fig. 2a) where the phases of the two fields have opposite signs.

Directional Propagation

We start by analyzing the simplest case of without EIT ($\Omega_c = 0$) shown in Fig. 3. When the probe is resonant $\Delta = 0$, both counter-propagating fields are equally detuned (with opposite signs) from the upper level by the Doppler shift (neglect recoil shift). If the probe is detuned $\Delta = -\omega_{\bar{p}} (< 0)$, the field that propagates opposite to the atoms would be absorbed since $G^- \simeq \kappa g w_{ab0} \mathcal{GL}$ is real and negative-value where $\mathcal{GL} = \int_{-\infty}^{\infty} \frac{e^{-x}}{\gamma^2 + x^2} dx$, $x = \omega_p - \omega_{\bar{p}}$ while the co-propagating field is transmitted since $G^+ \simeq -i\kappa g w_{ab0} \int \frac{f(p) dp}{\omega_{\bar{p}} + \omega_p}$ is primarily imaginary for $\gamma \ll \omega_{\bar{p}}$. The counter-propagating probe fields undergo optical rectification, i.e. one of the probes is transparent while the other probe field is absorbed. Thus, an appropriate probe field detuning with respect to the mean

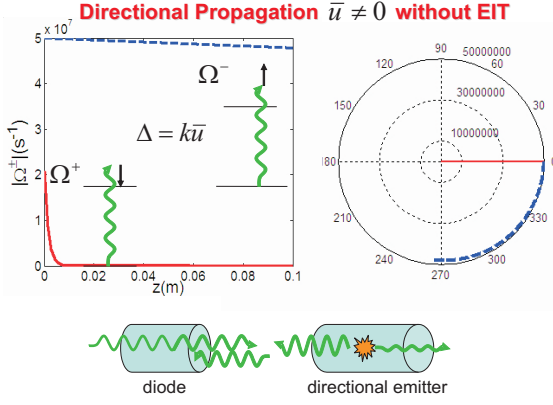


FIG. 3: (Color online) For positive detuning $\Delta > 0$ the field Ω^+ propagating to the right is red-shifted by atoms to resonant and heavily absorbed. The mean velocity is towards the right $\bar{u} > 0$. No control field is available. If the probe is passed through, the medium works like an optical diode. If the signal is generated from within the medium, it works like a directional emitter.

velocity of a gas jet can serve as an *optical diode*.

The mechanism may also be used as a *directional emitter* if an emitting source like an electrically driven quantum dot is embedded inside the channel of hollowed waveguide or fiber containing a gas flow. The source would emit light and would be guided along two opposite directions. The light in one direction would propagate with little damping while the light in the opposite direction is heavily absorbed.

Control of Directionality

Now, by applying a laser field Ω_c in EIT configuration, we can control the directional emission by controlling the absorption strength of the two fields. The field Ω_c splits the upper levels by $2\tilde{\Omega} = 2\sqrt{\Omega_c^2 - \gamma_{ac}^2}$, while the detuning which may add up or subtract with the Doppler shift. For $\tilde{\Omega} < \omega_{\bar{p}}$, the co-propagating field is damped more rapidly than the counter-propagating field because of its proximity to one of the split levels upon the Doppler shift. The reverse applies for $\tilde{\Omega} > \omega_{\bar{p}}$ (Figs. 4a and b). In the case of $\tilde{\Omega} = \omega_{\bar{p}}$, the two fields are equally detuned from the ac Stark shifted upper level.

Optical Transistor

In principle, the EIT scheme with stationary atoms works like an *optical valve*, with the control field acting as a knob that controls the intensity of the transmitted probe signal. In the absence of *atomic motion* or Doppler effect, both counter-propagating fields would be either equally damped or transparent but there is no *rectification*. The presence of atomic motion creates rectification of the probe signals, by damping one of the signals and inducing a transparency on the other signal. The ability of differential control of the two counter-propagating fields makes the device like an *optical transistor* as shown in Fig. 4c. The two signals to be rectified, transmitted or blocked through the flow of fluid and the control field.

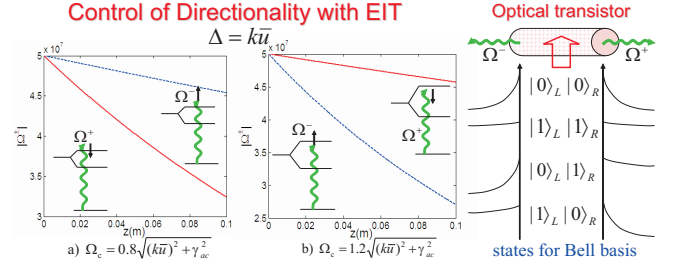


FIG. 4: (Color online) a) and b) show how one of the fields can be damped greater than the other by controlling the laser field. c) Optical transistor with two outputs produces four distinct states based on transparency and damping.

Quantum Velocimeter

An interferometer setup with one arm in a quantum coherence medium has been proposed as magnetometer [7]. We propose a new different setup (Fig. 5a) where *both* channels are in the phaseonium (EIT) medium and sensitive to Doppler effect. The interference of the counter-propagating fields can be used to detect atomic motion in a gas. The use of Doppler effect to measure the atomic velocity reminds us of the existing technique of Laser Doppler Velocimetry [8] which uses cross laser beam and Doppler effect to measure the flow velocity, a concept entirely based on classical physics. Here, we introduce *quantum velocimetry* that incorporates a different underlying mechanism, based on quantum coherence effect through the EIT with a laser as a control knob to create a sensitive velocimeter.

By combining the two probe fields as shown in Fig. 5a) the total field becomes

$$\Omega^{tot}(z) = \Omega^+(z) + \Omega^-(z) = \Omega_0(e^{G^+z} + e^{G^-z}) \quad (10)$$

The presence of atomic motion in the gas gives rise to $G^+ \neq G^-$. The real part gives the absorption or transparency of the field. The imaginary part corresponds to the linear susceptibility $\chi^\pm = gN\tilde{\rho}_{ab}(p, p \mp \hbar k, t)/\varepsilon_0\Omega^\pm$ and it gives oscillations or beating in $I(z) = |\Omega^{tot}(z)|^2$, as shown in Fig. 5a. The key point is that atomic motion can be detected through the presence of oscillations in $I(z)$ versus z , since there are no oscillations when $\bar{u} = 0$.

The sensitivity corresponds to the ability to detect a small change in velocity $\delta u = \frac{\Delta u}{\Delta \Omega_c} \delta \Omega_c$. From Fig. 5b, we estimate that $\frac{\Delta u}{\Delta \Omega_c} \sim 15/\gamma_{ac}$ which corresponds to $\Delta \omega_D = k\Delta \bar{u} \sim \Delta \Omega_c$. Thus, the classical sensitivity is

$$\delta u = \lambda \delta \Omega_c / 2\pi \quad (11)$$

which depends on the probe wavelength and the smallest detectable variation in the intensity. By taking $\delta \Omega_c \sim 3 \times 10^4 \text{ s}^{-1}$ as the smallest detectable value and $\lambda \sim 2 \times 10^{-6} \text{ m}$, we find $\delta u = 10 \text{ mm/s}$, which is enough to measure a flow as slow as the speed of an ant.

For $\Delta = 0$ Fig. 5c shows a valley or channel feature which decays exponentially at around $\Omega_c \simeq \omega_D = k\bar{u}$.

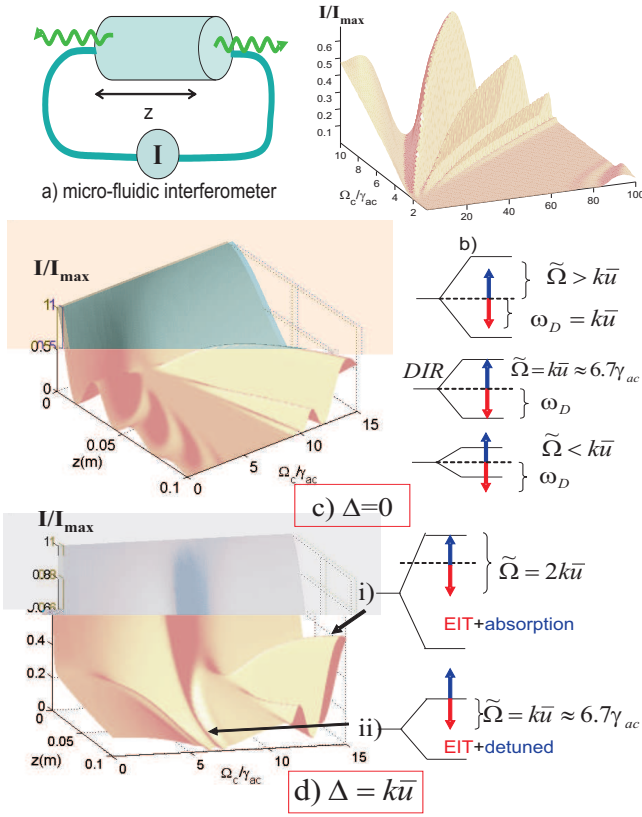


FIG. 5: (Color online) a) opto-fluidic interferometer, b) sensitivity of the device is found from the interference signal (normalized) versus the control field Ω_c and mean velocity \bar{u} at $z = 0.1m$. The mean velocity is obtained from the variation of the interference with z and Ω_c for c) $\Delta = 0$ and d) $\Delta = k\bar{u}$, where $\bar{u} = 100ms^{-1}$ corresponds to $\omega_D/\gamma_{ac} = 6.67$.

Here, the two fields are shifted to resonance and the imaginary part of G^\pm vanishes, so no oscillations. The probe fields add in the form $(e^{-bz} + e^{-bz})^2 \sim e^{-2bz}$. The channel divides the profile into two regions with oscillations. The oscillations in $\Omega_c < k\bar{u}$ are more rapid than the region $\Omega_c > k\bar{u}$. The fields in these regions interfere as $(e^{iaz}e^{-bz} + e^{-iaz}e^{-bz})^2 \sim e^{-2bz} \cos^2 az$.

For finite detuning (Fig. 5d), the ridge at $\Omega_c \simeq \omega_D$ corresponds to equal detuning (but opposite signs) of both probe fields from the ac Stark shifted level. One experiences EIT while the other is just normal detuning. We learn from Fig. 2b that their phases would be the same sign a superposition of EIT. So the fields interfere

as $|e^{-bz}e^{iaz} + e^{-bz}e^{-iaz}|^2 \rightarrow e^{-2bz}$, which explains the exponential decay. However, at $\Omega_c \simeq 2\omega_D$, we have one EIT and one resonant with the shifted upper level. Here, the fields interfere as

$$|e^{-bz}e^{iaz} + e^{-(b+c)z}|^2 = e^{-2bz} [4e^{-cz} \cos^2 \frac{az}{2} + (e^{-cz} - 1)^2] \quad (12)$$

where $c > 0$ which means that the damping due to absorption is larger than the damping in EIT. The rate of oscillations is less rapid by half. The rising ridge can be explained as due to the term $(e^{-cz} - 1)^2$ in Eq. (12).

Before we conclude, we briefly discuss a possible connection or analogy between the classical directional states with quantum states. For $\bar{u} = 0$, a resonant field with $\Omega_c = 0$ is damped, giving no-field state $|0\rangle_L|0\rangle_R$. However, a resonant field with finite Ω_c corresponds to transparency (EIT), thus the state of light is $|1\rangle_L|1\rangle_R$. For positive \bar{u} , when $\Delta = k\bar{u}$ co-propagating field experiences transparency whereas the counter-propagating field is absorbed, thus the state is $|0\rangle_L|1\rangle_R$, referred as "right-field". For $\Delta = -k\bar{u}$ and $\tilde{\Omega} = 2k\bar{u}$ or (for negative \bar{u} and $\Delta = k|\bar{u}|$) the situation is reversed giving the "left-field" state $|1\rangle_L|0\rangle_R$. The states are subjected to physical factors, namely the driving field and the direction of the fluid. If these four states (shown in Fig. 4c) can be used to construct the well-known Bell basis the scheme could be useful in quantum information. For example, the superposition of on and off control field can be described by a macroscopic entangled state

$$|\Phi^\pm\rangle = |0\rangle_L|1\rangle_R \pm |1\rangle_L|0\rangle_R \quad (13)$$

Similarly the superposition of "right-field" and "left-field" states is described by

$$|\Psi^\pm\rangle = |0\rangle_L|1\rangle_R \pm |1\rangle_L|0\rangle_R \quad (14)$$

which represents an indefinite mean velocity associated with the case of chaotic flow.

Finally, we conclude by noting that the full potential of optical directional control could be realized through miniaturization. The proposed optical transistor and velocimeter can be integrated with the existing microfluidic technology [9], creating a new class of *optical-microfluidic* sensor for further advances in chemical and biological sensing.

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